# Public Finance (First part)

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# Lecture 7:

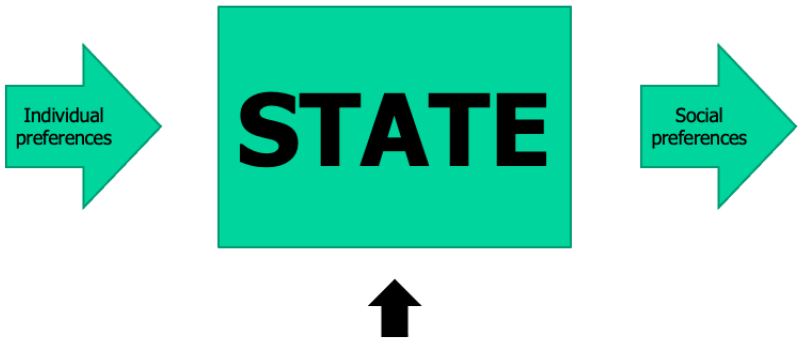
# Social choice theory

# Social choice theory

- Governments have several tools to intervene in the presence of market failures
- For instance, in the presence of extenalities
  1. Assigning property rights
  2. Pigovian tax
  3. Pigovian transfer
  4. Quantity regulation
  5. Emission fee

  ...
- Which one the government will choose? It depends...

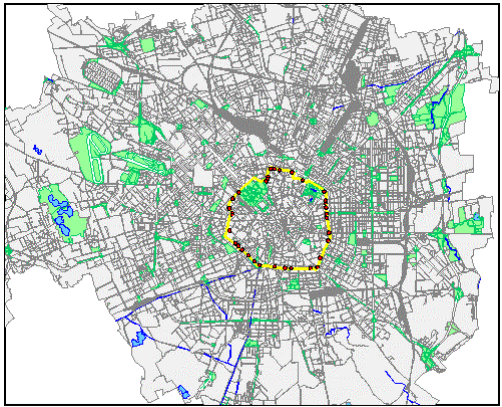
# Social choice theory



# Example: Milan congestion charge

- All vehicles (excluding electric and hybrid) that enter the city center of Milan must pay a 5-euro fee (Area C)
- This applies every time a vehicle crosses the city center borders
- Clear example of government intervention to address negative externality
  - Travelling by car has private benefits but pollutes the atmosphere
  - Travelling by car increases traffic for everybody else (other car users, public transportation users, bikers, pedestrians)
  - Buying an electic car has higher (private) costs but generates lower externalities
- Something in between a Pigovian tax and an emission fee
  - All access are taxed (not only beyond the efficient level)
  - Still, incentives to reduce emissions are present
    1. If you buy an electric car, there is no fee (even if you contribute to congestion)
    2. Municipality uses revenue to finance public transportation, bike sharing...

# Example: Milan congestion charge



- Area C is about 4.5% of the Milan's territory; 77K inhabitants

# Example: Milan congestion charge

- Is Area C the optimal way to address externality from car usage in a small and dense city centre? (normative question)
- We do not know, yet it has been implemented in 2012 and it is still in place
- Why? Area C is a politically feasible way to address externality from car usage in a small and dense city centre (positive question)
  - Approved by referendum
  - Survived a judiciary trial after appeal against it by owners of parking lots
  - Mantained by (as of now) several different administrations
  - Replaced a pure emission fee system in place until 2011 on which there was not enough political consensus

# Example: Milan congestion charge

- Many agents have stakes in the issue
  - Car owners
  - Municipality policy-makers
  - Retail stores and service providers
  - Residents of the restricted area
  - Public transportation users and pedestrians
  - ...
- To design effective policy, we cannot stop at looking for efficiency, we must also look at whether a certain policy is feasible in such a complex environment.

# Social choice theory

- In the introductory part of the course we have seen how a society can rank virtually any possible states of the world by defining a social welfare function
- For any states $A$ and $B$, we say that state $A$ is (strictly) preferred by the society compared to state $B$ if $W(A) > W(B)$
- Different societies may have different preference schemes and society's preference may as well change over time
- Defining a suitable SWF for a society is a complicated task!

# Social choice theory

- $K$ individuals have to take a collective decision over a set of $N$ alternatives options (*states of the world*)
- Each individual has rational preferences ranking the states of the world
- Each individual is different from each other in many dimensions, including constraints and preferences
- In order to rank states of the world according to a criterion, society needs to agree on a system suitable to aggregate individual preferences (i.e., on a social welfare function)

# Arrow's axioms

- Arrow proposes a number of axioms that a desirable aggregation mechanism of individual preferences should satisfy
- Arrow's axioms:
  1. **Unrestricted preference domain**
  2. **Weak Pareto principle**
  3. **No dictatorship**
  4. **Independence of irrelevant alternatives (IIA)**

# Arrow's axioms: Unrestricted preference domain

## Definition

A social choice satisfies the unrestricted preference domain axiom if any individual preferences that are rational (i.e., complete, symmetric, and transitive) are allowed and considered to compute the societal order. The social choice function that arises from aggregating these preferences should also satisfy rationality criterion.

- **Completeness**: It guarantees that collective choice is **effective**, that is, always able to determine which alternative should be chosen: if $\forall x \in X$ and $\forall y \in Y$ $x \succeq y$, or $y \succeq x$ then also the social choice must returns a definite output
- **Simmetry**: It guarantees that the order in which alternatives are presented should not matter. If $x \succeq y$, it should also be that $y \preceq x$.
- **Transitivity**: If $x \succeq y$ and $y \succeq z$, then $x$ must be (weakly) preferred over $z$.

# Arrow's axioms: weak Pareto principle

## Definition

A social choice function satisfies the (weak) Pareto principle if, for any pair of alternatives $x$ e $y$, the social choice prefers $x$ to $y$ if all individuals prefer $x$ over $y$.

- Notice the analogy with Pareto efficiency: social choice between $x$ and $y$ satisfies the weak Pareto principle if Society prefers the efficient outcome $x$ over the inefficient outcome $y$.
- According to the weak Pareto principle, the choice made by the collective preference relationship cannnot be Pareto dominated *among those possible*.
- Pareto principle does not limit in any way how a society should rank $x$ and $y$ if at least one individual prefers $y$ over $x$.

# Arrow's axioms: No dictatorship

## Definition

A social choice function satisfies the no dictatoriship axiom if the social choice does not coincide with the preferences of a single individual for any possible configuration of preferences of other individuals

- The criterion of collective choice must avoid that one individual, whatever the preferences of the others are, takes decision based only on own preferences
- Attention: axiom does not exclude the possibility that there is one decisive individual that swing a decision
- Notice also that axiom does not exclude the possibility of having 2 dictators. However, how would they aggregate together their own individual preferences?

# Arrow's axioms: Independence of irrelevant alternatives

## Definition

A social choice function respects the **Independence of Irrelevant Alternatives (IIA)** axiom when taking into account preferences for alternative $z$ does not affect the ranking order between $x$ and $y$.

- The axiom avoids strategic manipulation of the social choice (strategic voting, agenda-setting power)
- Example: Society prefers apple over peach
  1. Society prefers apple over orange over peach ✓
  2. Society prefers orange over apple over peach ✓
  3. Society prefers apple over peach over orange ✓
  4. Society prefers orange over peach over apple ×
  5. Society prefers peach over orange over apple ×
  6. Society prefers peach over apple over orange ×

# Does majority voting satisfy all Arrow's axioms?

## Definition

**Majority voting**: Mechanism used to aggregate individual votes into a social decision in which individual policy options are subject to a vote of all individuals and the option that receives the majority of votes is chosen

- **Majority voting** is certainly the most used mechanism for aggregating individual preferences, and therefore it is useful to understand what are its advantages and limits with respect to the Arrow's benchmark

# Majority voting

- Majority voting is easily applicable to a binary choice (i.e., **pairwise competition**). However, according to Arrow's unrestricted domain and independence of irrelevant alternatives axiom, choices are usually not restricted to two.
- Several alternative majority methods exists to rank among more than two choices:
  1. Simple majority (or plurality) rule
     - ⋆ Out of the many feasible alternatives, society chooses the one preferred by a relative majority of individuals
     - ⋆ No need that more than half individuals support the chosen alternative
  2. Runoff rules
     - ⋆ Out of the many feasible alternatives, if none is chosen by more than half of individuals, some alternatives are eliminated and individuals vote again until more than half of individuals support one choice

# Majority voting

3. System of pairwise comparisons
   - Individuals choose over each feasible pair of alternatives. The winner of all comparisons is selected by the society.
4. Sequential binary voting
   - Two of the many feasible alternative are selected to be voted in the first round. The most voted advance to the second and is put on vote against another alternative. So on until all feasible alternatives are put on vote
5. Rank system (Borda count)
   - Individuals rank preferences and assign a decreasing number of points to each of them. The alternative that scores the highest number of points is selected by the society

# Strategic voting

| type | preferences | numerosity |
|---|---|---|
| left | $a \succ b \succ c$ | 10 |
| center-left | $b \succ a \succ c$ | 8 |
| center-right | $b \succ c \succ a$ | 6 |
| right | $c \succ b \succ a$ | 20 |

- **Plurality:** if nobody "cheats", c prevails, but what if left votes for b?
- **Top-two runoff:** b prevails, unless the right moves 5 votes on a in the first round
- The outcome depends on the ability to coordinate and act strategically
  - Majority methods when more than two choices are available give incentives for strategic voting (i.e., IIA is violated)

# Majority voting and Condorcet's winner

## Definition

In majority voting, we define an alternative to be the **Condorcet winner** is it is the alternative that is chosen over any other in pairwise comparisons

- $x$ is the Condorcet winner if $x \succ y$ and $x \succ z$.
- Notice that the Condorcet winner may not be selected if other majority rules are applied.
  - $x$ might be the second best choice for those who prefer $y$ and the second best choice for those who prefer $z$, but it is not necessarily the first best choice of the plurality of voters
  - $x$ would win any second round in top-two runoff, but does not necessarily qualify

# Majority voting and Condorcet's winner

## Example

- Suppose there are 3 voters in a town: 1, 2, 3
- They have different preferences over the level of school spending (A, B, C)
  - Voter 1: $A \succ B \succ C$
  - Voter 2: $B \succ C \succ A$
  - Voter 3: $B \succ A \succ C$
- To reach a social choice, the town could proceed by voting sequentially on each pair of alternatives
  - Vote on funding level A vs B: **B wins**
  - Vote on funding level A vs C: **A wins**
  - Vote on funding level B vs C: **B wins**
- B has beaten both A and C and hence **B is the overall winner** (Condorcet Winner)
- **Majority voting has aggregated individual preferences** to produce a preferred social outcome.

# Majority voting and Condorcet's winner

- Which Arrow's axioms have we satisfied?
  - Unrestricted domain ?
  - Weak Pareto Principle ✓
  - No dictatorship ✓
  - Independence of irrelevant alternatives ✓

# Majority voting and Condorcet's winner

## Example

- Suppose there are 3 voters in a town: 1, 2, 3
- They have different preferences over the level of school spending (A, B, C)
  - Voter 1: $A \succ B \succ C$
  - Voter 2: $C \succ B \succ A$
  - Voter 3: $B \succ C \succ A$
- To reach a social choice, the town could proceed by voting sequentially on each pair of alternatives
  - Vote on funding level A vs B: **B wins**
  - Vote on funding level A vs C: **C wins**
  - Vote on funding level B vs C: **B wins**
- B has beaten both A and C and hence **B is the overall winner** (Condorcet Winner)
- **Majority voting has aggregated individual preferences** to produce a preferred social outcome.

# Majority voting and Condorcet's winner

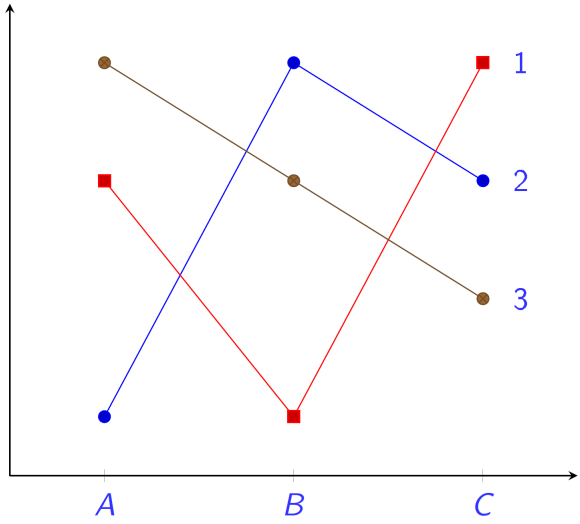
- Which Arrow's axioms have we satisfied?
  - Unrestricted domain ?
  - Weak Pareto Principle ✓(not violated)
  - No dictatorship ✓
  - Independence of irrelevant alternatives ✓

# Majority voting and NO Condorcet's winner

## Example

- There are three voters in a town: 1, 2, 3
- They have different preferences over the level of school spending (A, B, C)
  - Voter 1: $A \succ B \succ C$
  - Voter 2: $C \succ A \succ B$
  - Voter 3: $B \succ C \succ A$
- To reach a social choice, the town could proceed by voting sequentially on each pair of alternatives
  - Vote on funding level A vs B: **A wins**
  - Vote on funding level A vs C: **C wins**
  - Vote on funding level B vs C: **B wins**
- No alternative can beat the other two in a pairwise comparison. (Condorcet cycle)
- Majority voting has not aggregated individual preferences.

# Majority voting and NO Condorcet's winner




- Condorcet cycles are also known as the *Condorcet's voting paradox*

# Majority voting and NO Condorcet's winner

- Which Arrow's axioms have we satisfied?
  - Unrestricted domain $\times$
  - Weak Pareto Principle $\checkmark$
  - No dictatorship $\checkmark$
  - Independence of irrelevant alternatives $\checkmark$
- **Majority voting does not guarantee that unrestricted domain is satisfied when the other three axioms hold**

# Sequential majority voting

- **Sequential voting:** voting first on $A$ vs. $B$ and then voting on $C$ against the winner
  - **C wins** $\rightarrow$ unrestricted domain is satisfied
  - The choice depends on having $> 1$ individual supporting $A$ over $B$ and $> 1$ individual supporting $C$ over $A \rightarrow$ no dictatorship is satisfied
  - Weak Pareto principle is not violated
  - IIA is violated (order between $A$ and $B$ depends on whether $C$ is taken into account)
- Agenda setting power matters in reality
  - Voting on amendments before voting on the overall reform $\neq$ voting on a reform and then discussing amendments

**Sequential majority voting does not guarantee that IIA is satisfied when the other three axioms hold**

# Rank systems

- A single round of voting in which each voter expresses a preference on all the alternatives, and the one receiving the most votes is the winner
- **Pros**:
  - Unrestricted domain is satisfied
- **Cons**:
  - **IIA is not satisfied** and the incentive to strategic voting are concrete
- Let's analyze a specific case of rank system: the ***Borda count***
  - Each voter orders the alternatives by associating a value to each of them
  - If the alternatives are $N$, he will assign value $N$ to his favorite, $N-1$ to the second most favorite and so on until 1 that is assigned to the least vaorite one

# Borda count

## Example

| | $w$ | $x$ | $y$ | $z$ |
|---|---|---|---|---|
| Preferences of A | 4 | 3 | 2 | 1 |
| Preferences of B | 2 | 1 | 4 | 3 |
| Preferences of C | 1 | 4 | 3 | 2 |
| Total | 7 | 8 | 9 | 6 |
| Social choice (Borda) | 3° | 2° | 1° | 4° |

- Each voter orders the alternatives and assigns a decreasing value to them. The total for each alternative is used for selecting the social choice
- All alternatives are comparable, the ordering is transitive, no renunciation to any kind of preferences is required

# Borda count

## Example

- We can see that the relative ranking of $x$ and $y$ is not independent on the presence/absence of alternatives $w$ and $z$)
  - Individual 1: $w \succ x \succ y \succ z$
  - Individual 2: $y \succ z \succ w \succ x$
  - Individual 3: $x \succ y \succ z \succ w$
- Individual 1 and individual 3 prefer $x$ over $y$. However, $y$ is preferred by society because $x$ gets only 1 point for individual 4.

# Borda count

## Example

| | $w$ | $x$ | $y$ |
|---|---|---|---|
| Preferences of A | 3 | 2 | 1 |
| Preferences of B | 2 | 1 | 3 |
| Preferences of C | 1 | 3 | 2 |
| Total | 6 | 6 | 6 |
| Social choice (Borda) | ? | ? | ? |

- **Suppose to drop $z$**
- All choices get the same overall preference: **the social choice is not independent from the irrelevant alternative $z$**. By adding it we arrive to a choice, by removing it we have indecision

# Arrow's impossibility theorem

**Theorem**

*Any social choice mechanism which satisfies the unrestricted domain, IIA and Pareto principle axioms must be dictatorial.*

- Not only majority voting, but any voting system we may think of cannot satisfy all ethical criteria of aggregation specified by Arrow.
- **Is dictatorship good? Not at all!** But we have to bear in mind that non-dictatorial systems need to put limits on at least one of the other three axioms.

# Arrow's impossibility theorem

- Arrow's impossibility theorem does not state it is impossible to find a social choice mechanism that works. It only postulates that it is impossible to find a social choice mecanism that works under all circumstances.
- Allowing limited violations of either the unrestricted domain axiom or of the IIA axiom may still help
  - For instance, majority voting may work if we exclude the case of Condorcet's cycles
- Is it preferable to limit the feasible set of preferences or not to value IIA?

# Restricting preference domain

- Relaxing IIA is more problematic than it looks
  - It would require to think of preferences as cardinal and comparable across individuals
  - Example: rank system. Why should we postulate that the distance between most preferred and second-best alternative has to be the same for all individuals?
- Relaxing unrestricted domain is instead less problematic than it seems
  - Not to put limits on individual freedom is valuable
  - However, Arrow's impossibility result depends on the fact that also the most incredible preferences should be admitted

# Restricting preference domain

- Consider the previous example, and say $A$, $B$, $C$ are levels of public schooling investment
  - A is high; B is middle; C is low
- Let us consider the unrestricted preference domain
  1. $A \succ B \succ C$
  2. $A \succ C \succ B$
  3. $B \succ C \succ A$
  4. $B \succ A \succ C$
  5. $C \succ A \succ B$
  6. $C \succ B \succ A$
- (2) and (5) appear inconsistent with how we usually think of utility functions (unimodal)

# Median voter theorem

- In the example, we are making two simplifications w.r.t. Arrow's criteria
  - $A$, $B$, $C$ can be ordered along the same (monetary) dimension
  - Excluding (2) and (5) implies that preferences are restricted to be unimodal
- When these two additional requirements are fulfilled, it is possible to show that there always exists a Condorcet winner

## Theorem

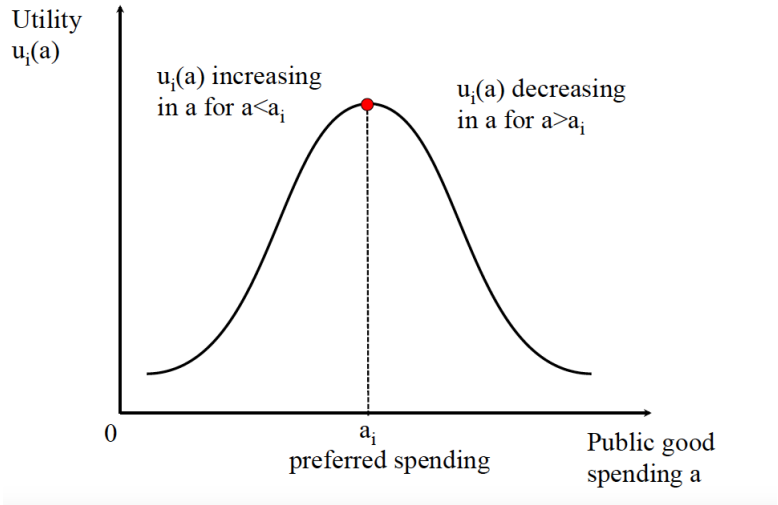
*If voters' preferences are **unimodal** on a single-dimensional space, then:*

- *The **social choice function** emerging with majority vote **is transitive**. In other words, there always exists a Condorcet winner*
- ***The Condorcet winner** is the alternative preferred by the median voter*
- *The median voter is the individual such that the peak of (K-1)/2 are not above the peak of the median voter and the peak of (K-1)/2 individuals are not below the peak of the median voter.*

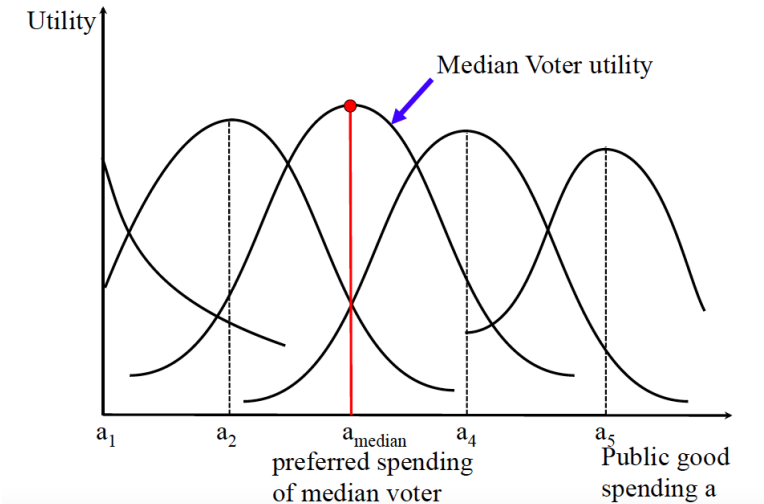
# Median voter theorem

- The intuition behind the median voter theorem is simple and straightforward:
  - If we can rank individuals according to their preferences along a line, there will be (K-1)/2 individuals who prefer the median voter's preferred choice than the other (K-1)'s individuals to the other side of her
  - In turn, any alternative which is not the preferred choice of the median voter will not receive the support of (at least) (K-1)/2 voters to either the left or the right of the median voter, plus the median voter.

# Median voter theorem

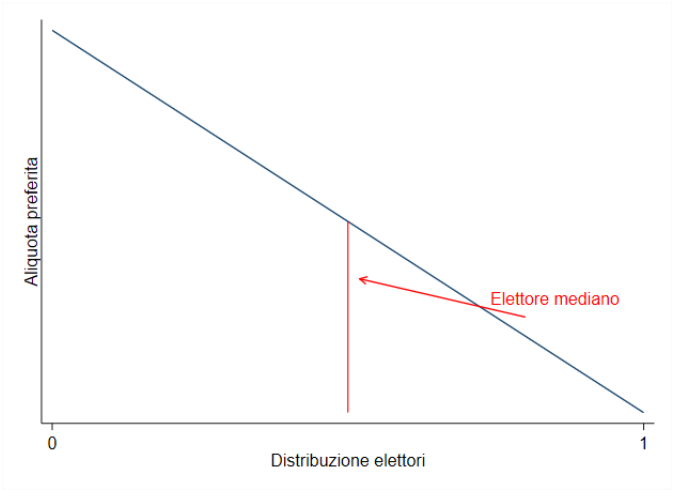


# Median voter theorem



# Median voter theorem: remarks

- MVT applies $\forall N > 2$
- MVT applies to both discrete and continuous sets of alternatives
- The condition of the MVT are sufficient but not necessary
- When the individual indifference curves are convex, then the preferences are always unimodal
  - **Standard utility functions are such that indifference curves are always convex**
- Notice: even if the median voter is basically a dictator (her preferred option will be implemented), the theorem does not mean that majority voting violates the no dictatorship axiom
  - Whether an individual or another is the median voter depends on the preferences of all individuals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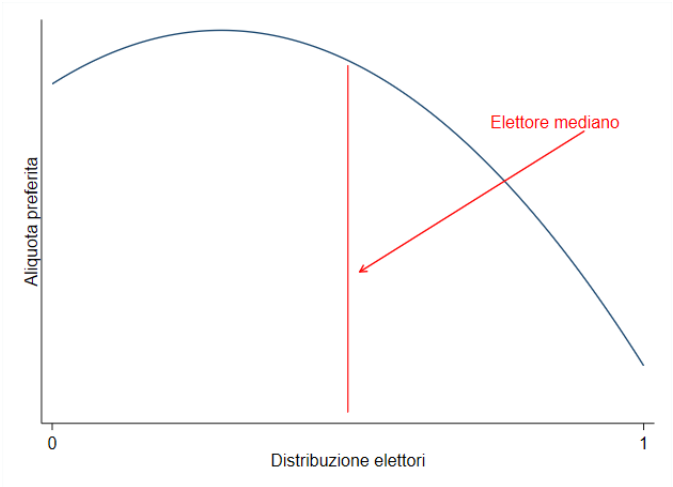
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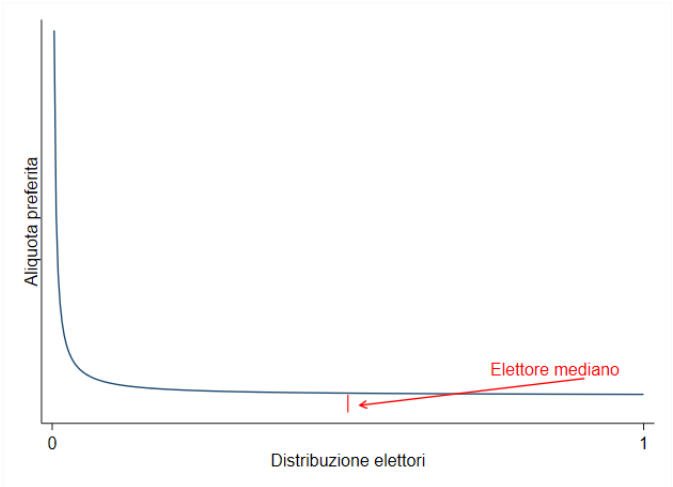
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