Public Finance Exercises

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Note

This file contains two examples of each exercise category that may be included in the mid-term exam on May 17$^{\text{th}}$, 2022 or in the final exams (for what concerns the first module of the course). Please notice that, i) in the exams, you will be also asked open questions on the course material, ii) exercises in the exam will feature small modifications as compared to the ones here below, and iii) these exercises are just a learning tool and solving them is not compulsory. Solutions will not be provided.

1 Externalities

1. Consider two individuals $i = \{1, 2\}$ who can buy shotguns (S_i) to go hunting for birds. Each shotgun costs c. The number of birds that a shotgun can hunt is an increasing and concave function of the total number of shotguns bought by the two individuals $F(S)$, where $S = S_1 + S_2$. For the sake of the exercise, let us assume that $F(S) = 2\sqrt{S}$.

   (a) Calculate the private optimal number of shotguns bought by individual 1 and individual 2. Calculate the aggregate number of shotguns bought by the society if individuals take their decision privately.

   (b) Calculate the efficient number of shotguns.

2. Consider two firms $i = \{1, 2\}$ who can build platforms (P_i) to extract fuel oil. Each platform costs c. The number of gallons that a platform can extract is an increasing and concave function of the total number of platforms built by the two companies $F(P)$, where $P = P_1 + P_2$. For the sake of the exercise, let us assume that $F(P) = 10\sqrt{\frac{S}{2}}$.

   (a) Calculate the private optimal number of platforms bought by firm 1 and firm 2. Calculate the aggregate number of platforms build by the society if firms take their decision privately.

   (b) Calculate the efficient number of platforms.

2 Public goods

1. Two individuals $i = \{1, 2\}$ have equal income $Y_i = 100$ which can be used in part to purchase shirts and in part to contribute to the provision of policemen. For the sake of the exercise, assume utility of individual i to be $U_i = 3log(s_i) + 6log(P)$, where s_i is the number of shirts purchased by individual i and $P = P_1 + P_2$ is the total number of policemen hired by a society whose members are individual 1 and individual 2.

   (a) Calculate the optimal private choices of s_1, s_2, P_1, P_2.

   (b) Calculate the efficient society's choice according to the Samuelson rule (you can decide whether to apply the rule or to solve the aggregate society's problem).

2. Two countries $i = \{1, 2\}$ have equal revenues $Y_i = 5$ which can be used in part to provide hospitals for their own citizens and in part to contribute to a joint space mission by purchasing

common shuttles. For the sake of the exercise, assume utility of country i to be $U_i = \sqrt{h_i S}$, where h_i is the number of hospitals built by country i and $S = S_1 + S_2$ is the total number of shuttles provided by the two countries.

(a) Calculate the optimal private choices of h_1, h_2, S_1, S_2.

(b) Calculate the efficient society's choice according to the Samuelson rule (you can decide whether to apply the rule or to solve the aggregate society's problem).

3 Social choice

1. Consider a society composed of three individuals $i = \{1, 2, 3\}$ with preferences over three goods apple, banana, and cherry equal to

   - Individual 1: $a \succ b \succ c$
   - Individual 2: $a \succ c \succ b$
   - Individual 3: $b \succ c \succ a$

   and suppose the society can only buy one of the three goods.

   (a) Which good is the Condorcet's winner?

   Suppose instead that preferences are such that

   - Individual 1: $a \succ b \succ c$
   - Individual 2: $c \succ a \succ b$
   - Individual 3: $b \succ c \succ a$

   (b) Which good is the Condorcet's winner?

   Suppose now that society first chooses (vith majority voting) between apple and banana, and then the more preferred alternative is voted against cherry (with majority voting).

   (c) Which good will the society choose?

2. Consider a society composed of 100 individuals, with preferences:

   - Left: $a \succ b \succ c \rightarrow n_l = 25$
   - Centre-left $b \succ a \succ c \rightarrow n_{cl} = 20$
   - Centre-right $b \succ c \succ a \rightarrow n_{cr} = 15$
   - Right: $c \succ b \succ a \rightarrow n_r = 40$

   (a) Calculate which choice is preferred by the society under simple majority voting if all individuals vote sincerely.

   (b) Calculate which choice is preferred by the society under top-two runoff voting if all individuals vote sincerely.

   (c) Calculate which choice is preferred by the society under a Borda type of voting in which each individual assigns 2 points to the most preferred alternative, 1 point to the second-best, and 0 points to the least preferred alternatives and all individuals vote sincerely.

   (d) Make one example (based on a voting system of your choice) on how groups that support losing alternatives may act strategically to avoid the expected results.

4 Political economy

1. In a society of 10 individuals there are two political candidates, L (*Left*) and R (*Right*) that compete for office under a majority rule. Voters belong to two different groups: working-class and upper-class. 7 individuals belong to the working class and 3 individuals belong to the upper class. Each voter $i \in \{1, ...10\}$ has quasi-linear utility function: $U_i = c_i + ln(G)$, where c_i is private consumption and G is a public good. The individual budget constraint is $c_i = (1-t)w_i$, where w_i represents individual income, and t is the tax rate. All voters belonging to a group have the same income. In particular, all individuals belonging to the working class have $w_i = 3$ and all upper class voters have $w_i = 9$. The government budget constraint is $G = 7 \times 3 \times t + 3 \times 9 \times t$. The politicians propose t and G before the election, and must commit on their proposal.

   (a) Which tax rate, t and level of public good G, will office motivated candidates propose prior to the election?

   (b) Suppose now that different groups have different voting turnout. All voters in the upper class vote and each individual of the working class has a probability $\frac{1}{3}$ of voting. Assume again $\alpha = 0.3$ and $\beta = 0.1$. Solve for tax rate and the level of public good.

   (c) Calculate what would be the efficient level of t and G according to an utilitarian SWF.

2. In a parliamentary democracy, three parties are in the House of Representatives. We refer to them as c (*Center*), l (*Left*) and r (*Right*). None of them got the majority during the elections, but a coalition of two is necessary and sufficient to form a government. Parties can bargain over the amount of resources allocated from the public budget $R = 100$ to the working class (represented by the Left party), the middle class (represented by the Center party), and the upper class (represented by the Right party). Utility of party p is equal to $U_i = r_p$ where r_p is the amount of resources that the budget assigns to the group represented by party p. Assume that the timing of the events is the following: one party is randomly chosen (with probability $p_p = \frac{1}{3}$ equal for all parties) as the first proponent. If a coalition of two parties is formed, parties receive payoffs accordingly. Otherwise, a new party is randomly chosen (with probability $p_p = \frac{1}{3}$ equal for all parties) as a new proponent. If a coalition is formed, parties receive payoffs accordingly. If no coalition is formed, all parties get a payoff of 0.

   (a) What would be the distribution of resources if the game arrives to the last period?

   (b) Calculate the unique Subgame-perfect Nash Equilibrium of this game.