Public Finance (First part)

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Lecture 2 and Lecture 3:
Review of theoretical tools

Theoretical and Empirical Tools

- **Theoretical tools**: The set of tools designed to understand the mechnisms behind the economic decision making
  - **Example**: Economists model individuals' choices using the concepts of utility function maximization subject to budget constraint
  - **Drawback**: Narrow view of human behavior. It works well for consumption choices but we need to make simplifying assumptions
- **Empirical tools**: The set of tools designed to analyze data and answer questions raised by theoretical analysis

Utility Mapping of Preferences

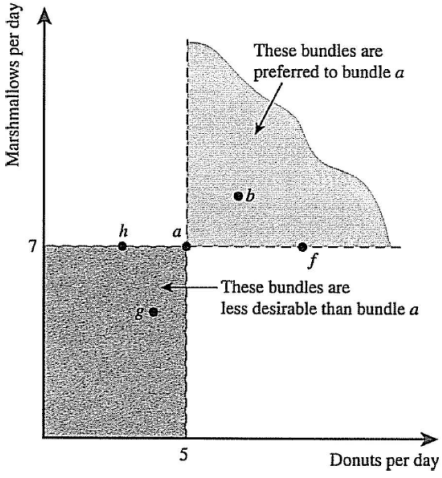
Definition

Utility function: A mathematical function translating a human action (typically consumption) into specific level of personal utility:

$$U = u(X_1, X_2, X_3, ...)$$

where $X_1, X_2, X_3, ...$ are the quantity of goods 1,2,3,... consumed by the individual

- In these slides we will consider a utility depending on two goods only $u(X_1, X_2)$
- **Example**: $u(X_1, X_2) = \sqrt{X_1 \cdot X_2}$ with X_1 number of donuts and X_2 number of marshmallows
- Key assumption: individual utility **increases** with the level of consumption of each good.


Marshmallows per day
These bundles are
preferred to bundle a
b
h
a
7
f
These bundles are
less desirable than bundle a
g
5
Donuts per day

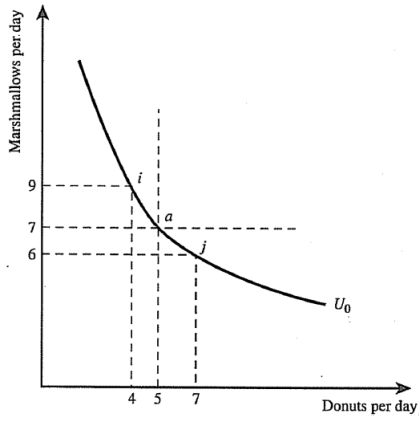
Indifference curves

Definition

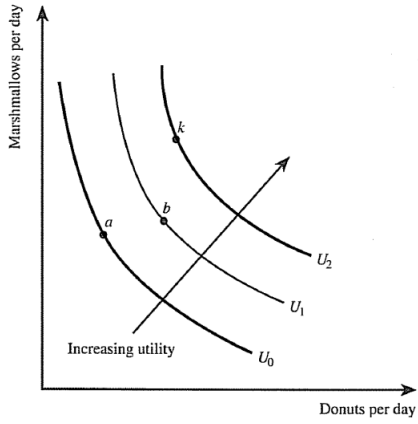
A graphical representation of all bundles of goods that make an individual equally well off

Mathematically: the indifference curve giving an utility level $\bar{U}$ is given by the set of bundles (X_1, X_2) such that $u(X_1, X_2) = \bar{U}$

- Indifference curves have two essential properties, both of which follow naturally from the more-is-better assumption:
  1. Indifference curves are always downward sloping (to keep utility fixed, when one of the goods consumed increases the other must be reduced).
  2. Consumers prefer higher indifference curves (the bundles in a higher indifference curves have non-lower amounts of goods).



$$U(a) = U(j) = U(i) = U_0$$


Marshmallows per day
k
b
a
U_2
U_1
U_0
Increasing utility
Donuts per day

$$U(a) < U(b) < U(k)$$

Marginal utility

Definition

The increment of utility obtained by consuming an additional unit of a good. The marginal utility of good 1 is defined as:

$$MU_1 = \frac{\partial u}{\partial X_1} \simeq \frac{u(X_1 + dX_1, X_2) - u(X_1, X_2)}{dX_1}$$

It is the derivative of utility with respect to X_1 keeping X_2 constant (**partial derivative**).

- Utility functions exhibit **diminishing marginal utility**. $\partial u / \partial X_1$ is positive but decreasing in X_1: the consumption of each additional unit of a good gives less extra utility than the consumption of the previous unit.

$$\frac{\partial u}{\partial X_1} > 0, \quad \frac{\partial^2 u}{\partial X_1^2} < 0$$

Marginal rate of substitution (MRS)

Definition

The marginal rate of substitution between good 1 and good 2 is the rate at which the consumer will trade good 2 to increase the consumption of good 1.

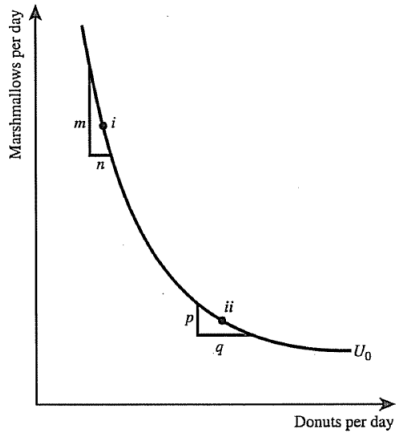
Marginal rate of substitution between good 1 and good 2 is:

$$MRS_{1,2} = \frac{MU_1}{MU_2}$$

- Individual is indifferent between 1 unit of good 1 and $MRS_{1,2}$ units of good 2.

$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow MRS_{1,2} = \frac{X_2}{X_1}$$

- Graphically, $MRS_{1,2}$ is equal to (minus) the slope of the indifference curve.



Near bundle *ii*, the loss in utility from reducing consumption of donuts by q units is offset by the increase in utility from increasing consumption of marshmallows by p units.

Budget constraint

Definition

A mathematical representation of all the combinations of goods affordable by individuals if they spend their entire income.

$$p_1 X_1 + p_2 X_2 = Y$$

with p_i price of good i, and Y disposable income.

- Budget constraint defines a linear set of bundles the consumer can purchase with its disposable income Y

$$X_2 = \frac{Y}{p_2} - \frac{p_1}{p_2} X_1$$

- The slope of the budget constraint is $-p_1/p_2$.
- If the consumer gives up 1 unit of good 1, she has p_1 more to spend, and can buy p_1/p_2 units of good 2.

Utility maximization

- Individual maximizes utility subject to budget constraint:

$$\max_{X_1, X_2} u(X_1, X_2) \quad \text{subject to} \quad p_1 X_1 + p_2 X_2 = Y$$

$$\textbf{Solution:} \quad MRS_{1,2} = \frac{p_1}{p_2}$$

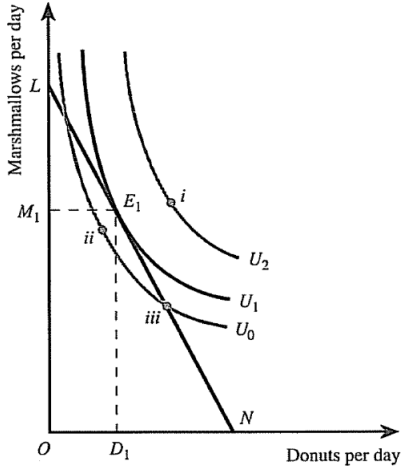
Proof: Budget implies that $X_2 = (Y - p_1 X_1)/p_2$

Individual chooses X_1 to maximize $u(X_1, (Y - p_1 X_1)/p_2)$

The first order condition (FOC) is:

$$\frac{\partial u}{\partial X_1} - \frac{p_1}{p_2} \cdot \frac{\partial u}{\partial X_2} = 0.$$

- At the optimal choice, the individual is indifferent between buying 1 extra unit of good 1 for \$ p_1 and buying p_1/p_2 extra units of good 2 (also for \$ p_1).

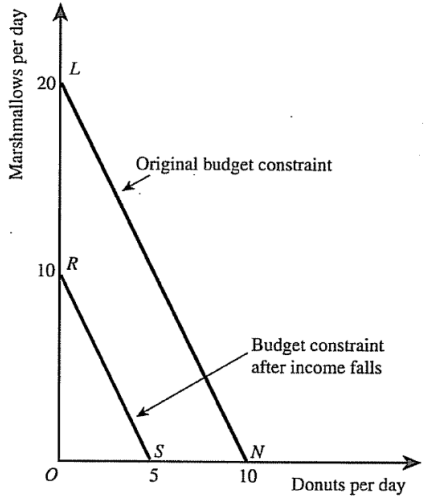

Marshmallows per day
L
M_1
E_1
i
ii
iii
U_2
U_1
U_0
N
O
D_1
Donuts per day

Income and substitution effects

- Individual maximization generates demand functions $X_1(p_1, p_2, Y)$ and $X_2(p_1, p_2, Y)$
- Let us see how much does $X_1(p_1, p_2, Y)$ vary with p and Y

1. Change in demand due to a change in Y is pure **income** effect.
   - If Y goes down, the individual is poorer and reduces consumption of both goods.
2. Change in demand due to a change in p is **price** effect.
   - If p_1 goes up, good 2 becomes relatively cheaper. The consumer may want more of it and less of good 1.
   - If p_1 goes up, the individual is poorer because she has to pay more for the units of good 1 she decides to consume. In turn, consumption of good 2 may be also reduced even if more convenient.
   - Price effect made of two components: an **income** effect and a **substitution** effect.

Income effect

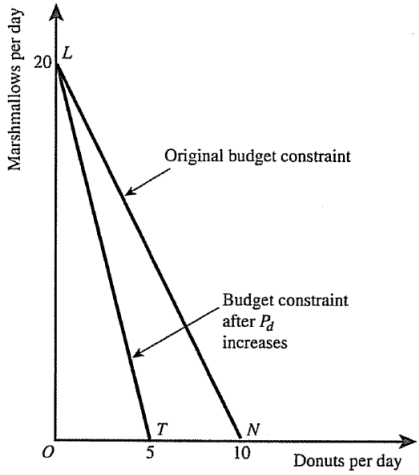


Budget constraint - Effect of a reduction in income

Normal and inferior goods

- Income effects are different for different goods and might be positive or negative depending on whether the demand for a good increases or decreases due to a variation in income.
- **Normal goods**: Goods for which demand increases as income Y rises: $X_1(p, Y)$ increases with Y (most goods are normal)
- **Inferior goods**: Goods for which demand falls as income Y rises: $X_1(p, Y)$ decreases with Y (examples: public transportation, cheap food)

Price effect



Budget constraint - Effect of an increase in donut price

Decomposition of the price effect

- A change in p_1 affects the slope of the budget constraint and can be decomposed into 2 effects:
  - **Substitution effect**: Holding utility constant, a relative rise in the price of a good will always cause an individual to choose less of that good
  - **Income effect**: A rise in the price of a good will typically cause an individual to choose less of all goods because her income can purchase less than before
- For normal goods, an increase in p_1 reduces $X_1(p_1, p_2, Y)$ through both substitution and income effects. An increase in p_1 may increase $X_2(p_1, p_2, Y)$ depending on whether income or substitution effect prevails.

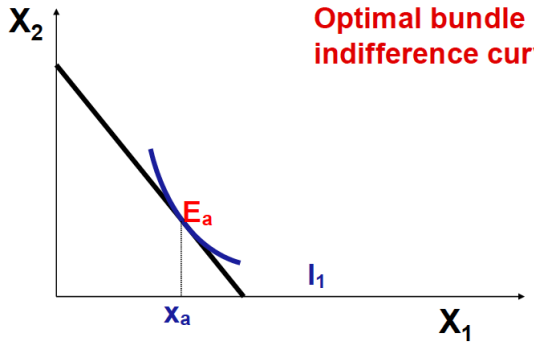
Decomposing the price effect: Hicksian method

- To isolate the substitution effect we ask:

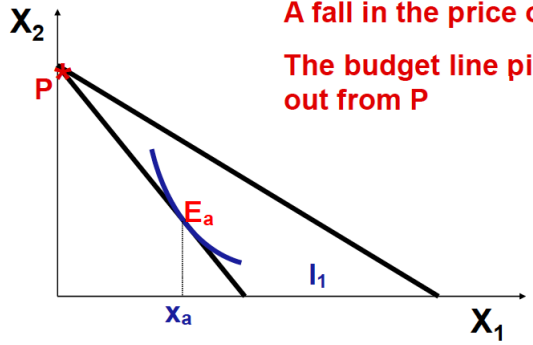
  "what would the consumer's optimal bundle be if s/he faced the new lower price for X1 but experienced no change in real income?"

- This amount would return the consumer to the original indifference curve (I_1)

Decomposing the price effect: Hicksian method



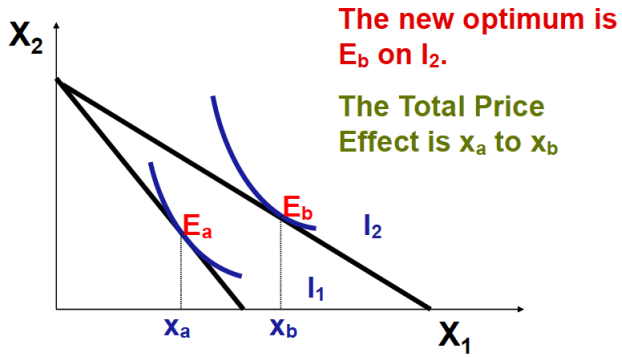
Decomposing the price effect: Hicksian method



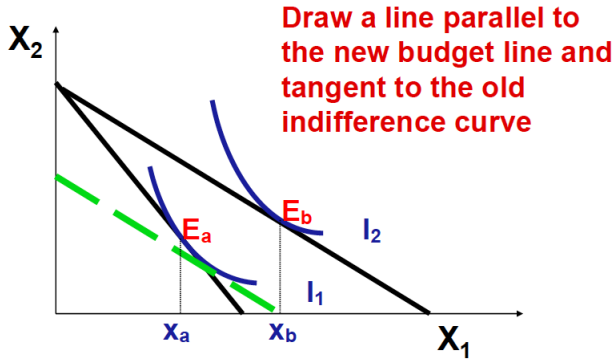
A fall in the price of X_1

The budget line pivots out from P

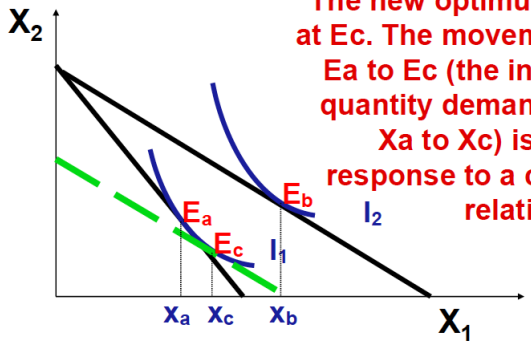
Decomposing the price effect: Hicksian method



Decomposing the price effect: Hicksian method

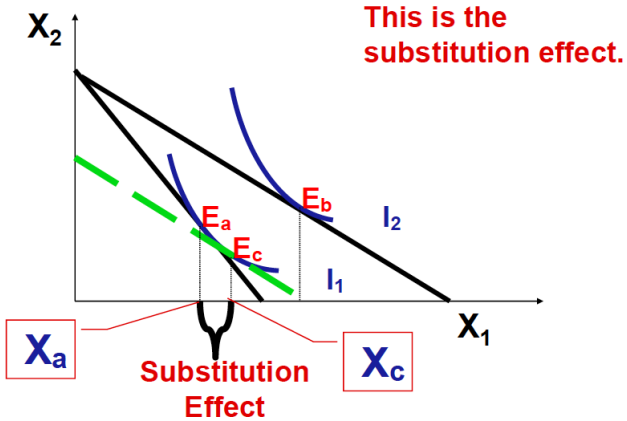


Decomposing the price effect: Hicksian method

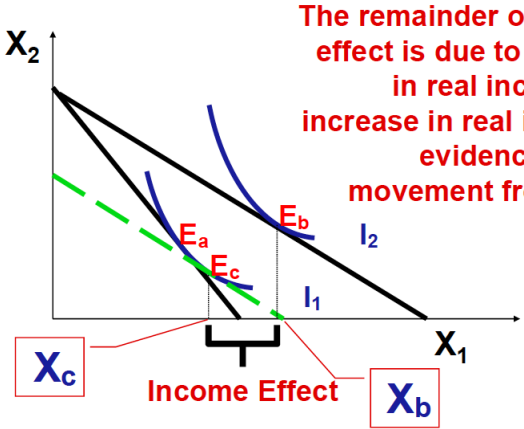


The new optimum on I_1 is at Ec. The movement from Ea to Ec (the increase in quantity demanded from Xa to Xc) is solely in response to a change in relative prices

Decomposing the price effect: Hicksian method

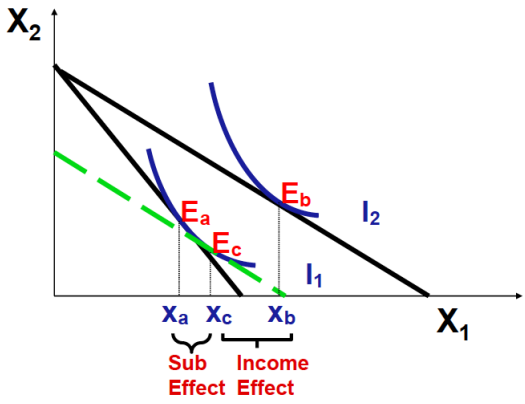


Decomposing the price effect: Hicksian method



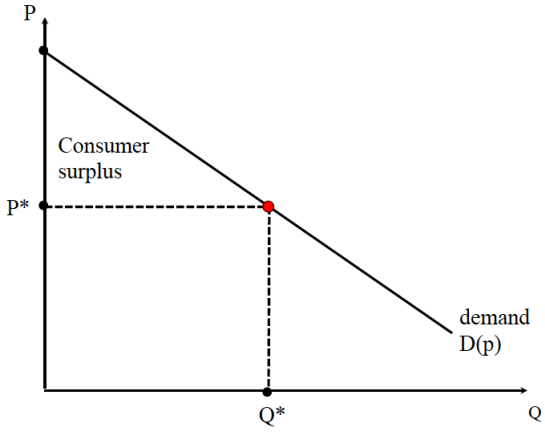
The remainder of the total effect is due to a change in real income. The increase in real income is evidenced by the movement from I_1 to I_2

Decomposing the price effect: Hicksian method



Consumer surplus

- At price p, demand is $D(p)$ and p is the \$ value for consumer of the marginal (last) unit consumed
- First unit consumed generates higher marginal utility than the price necessary to afford it and hence a surplus, last (marginal) unit consumed generates no surplus because price equalizes marginal utility.
- Consumer surplus can be measured as area below the demand curve and above the price horizontal line


P
Consumer
surplus
P*
demand
D(p)
Q*
Q

Price elasticity of demand

Definition

The % change in demand caused by a 1% change in the price of that good:

$$\varepsilon^D = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}} = \frac{\Delta D/D}{\Delta p/p} = \frac{p}{D}\frac{dD}{dp}$$

- Elasticities can be used effectively in comparisons because they are **unit free**
- $\varepsilon^D = pD'(p)/D(p)$ is a function of p and hence can vary with p along the demand curve

Properties of the elasticity of demand

1. Typically negative, since quantity demanded typically falls as price rises.
2. Typically not constant along a demand curve.
3. With vertical demand curve, demand is **perfectly inelastic** ($\varepsilon = 0$). Demanded quantity of good does not depend on price.
4. With horizontal demand curve, demand is **perfectly elastic** ($\varepsilon = -\infty$). A marginal reduction in price moves consumption from 0 to ∞.
5. The effect of one good's prices on the demand for another good is the **cross-price** elasticity. Typically, not zero.

Producers and profit maximization

- Producers (typically firms) use technology to transform inputs (labor and capital) into outputs (consumption goods)
- Goal of producers is to maximize profits = sales of outputs minus costs of inputs
- Production decisions define supply functions
  - Profits $\Pi(p, Q) = p \cdot Q - c(Q)$ where $c(Q)$ is cost of producing quantity Q. $c(Q)$ is decreasing and convex ($c'(Q) < 0$ $C''(Q) > 0$). In competitive markets, firms take p as given.
  - Profit maximization: $\max_Q [p \cdot Q - c(Q)]$
    $\Rightarrow c'(Q) = p$: marginal cost of production equals price
  - Defines the supply curve $Q = S(p)$.

Supply curve

Definition

Supply curve $S(p)$ is the quantity that firms in aggregate are willing to supply at each price: typically upward sloping because marginal costs are decreasing in supplied quantity

- At price p, producers produce $S(p)$, and the \$ cost of producing the last unit is p

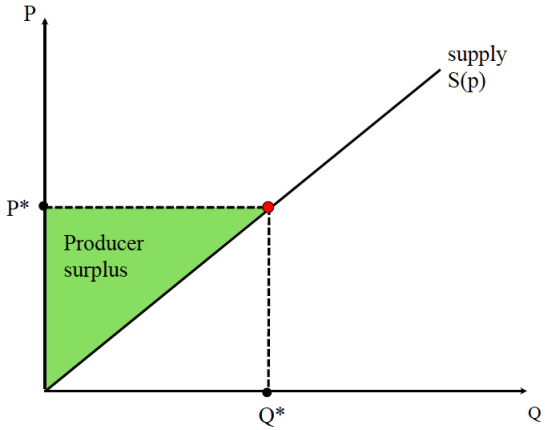
Definition

Elasticity of supply ε_S is defined as

$$\varepsilon_S = \frac{\%\text{ change in quantity supplied}}{\%\text{ change in price}} = \frac{\Delta S/S}{\Delta p/p} = \frac{p}{S}\frac{dS}{dp}$$

$\varepsilon^S = pS'(p)/S(p)$ is a function of p and hence can vary with p along the supply curve

Producer surplus



Market for goods

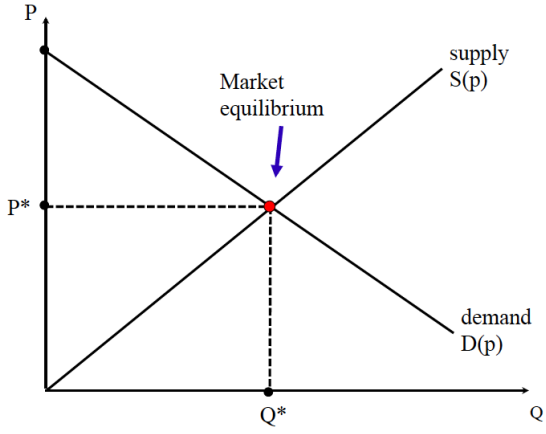
- Consumers (demand) and producers (supply) interact on markets for goods.
- The market we are considering relies on a set of assumptions:
  1. All agents have a price-taking behaviour - **perfect competition** (no single consumer or producer can exert market power)
  2. All agents have **perfect information** on all aspects relevant to the trade
  3. All agents are **perfectly rational** (capable to evaluate correctly all the available information)
  4. There are no **transaction costs** (interacting in the market does not entail any fee, wasted time, etc)
  5. It is a **spot market**, all the relevant information for the trade is embodied in price and quantity (no externalities, no interaction apart from the trade ones, e.g., no social bonds)

Market equilibrium

Definition

The equilibrium is the price p^* such that $D(p^*) = S(p^*)$

- In the simple diagram, p^* is unique if $D(p)$ decreases with p and $S(p)$ increases with p
- If $p > p^*$, then supply exceeds demand, and price needs to fall to equilibrate supply and demand
- If $p < p^*$, then demand exceeds supply, and price needs to increase to equilibrate supply and demand.

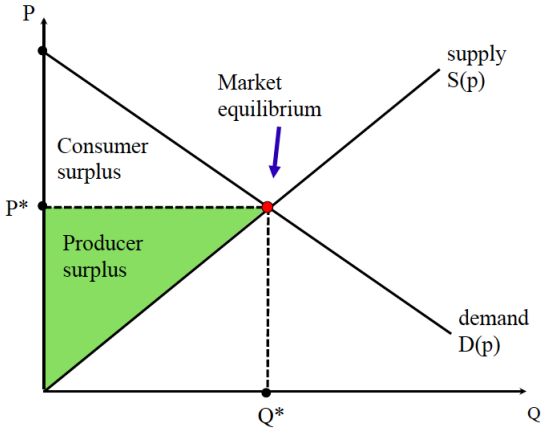

P
supply
S(p)
Market
equilibrium
P*
demand
D(p)
Q*
Q

Aggregate surplus

Definition

Aggregate surplus represents the net gains to society from all trades that are made in a particular market, and it consists of two components: consumer and producer surplus.

- **Consumer surplus**: The benefit that consumers derive from consuming a good, above and beyond the price they paid for the good. It is the area below demand curve and above market price.
- **Producer surplus**: The benefit producers derive from selling a good, above and beyond the cost of producing that good. It is the area above supply curve and below market price.
- **Aggregate surplus**: The sum of consumer surplus and producer surplus. It is the area above supply curve and below demand curve.

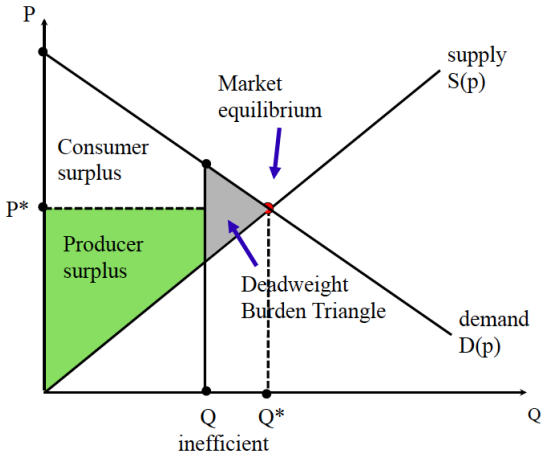

P
supply
S(p)
Market
equilibrium
Consumer
surplus
P*
Producer
surplus
demand
D(p)
Q*
Q

How does the market leads agents to equilibrium?

- The market is driven to equilibrium by the rational choices of agents aiming to maximize their utility
- When considering a trade market, utility is directly related to agents surplus $\Rightarrow$ if we define **consumer/producer marginal surplus** as the surplus arising by increasing the quantity traded by one unit we can note that:
  - If $Q < Q^*$ the **sum of marginal consumer and producer surplus is positive** so quantity traded would increase (and so would price)
  - If $Q > Q^*$ the **sum of marginal consumer and producer surplus is negative** so quantity traded would decrease (and so would price)
  - At $Q = Q^*$ **both marginal consumer and producer surplus are zero** so, irrespective from the approaching direction, the quantity and prices will not change.

Competitive equilibrium maximizes social welfare

- The competitive equilibrium where supply equals demand, maximizes total economic surplus
- Economic surplus just counts dollars regardless of who gets them ($1 to rich producer better than $.99 to poor consumer) $\Rightarrow$ blindness to distributional aspects
- **Deadweight loss**: The reduction in economic surplus from denying trades for which benefits exceed costs when quantity differs from the efficient quantity
  - Key rule: Deadweight loss triangle points to the efficient allocation, and grows outward from there


P
supply
S(p)
Market
equilibrium
Consumer
surplus
P*
Producer
surplus
Deadweight
Burden Triangle
demand
D(p)
Q
Q*
Q
inefficient

Pareto efficiency

Definition

Production is Pareto-efficient (or optimal) when it is not possible to reallocate the production factors to produce additional units of a good without decreasing the quantity produced of the other good.

Definition

Exchange is Pareto-efficient (or optimal) when it is not possible to reallocate the goods X and Y to increase the utility of one agent without decreasing the utility of the other agent

1st Theorem of Welfare Economics

Theorem

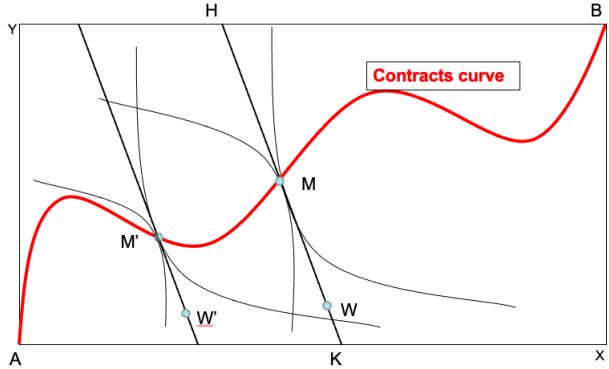
If conditions 1–5 (from slide 29) are met, every competitive market equilibrium is Pareto efficient

↓

How can Government's intervention be justified?

Why does the government intervene?

- Pareto efficiency is desirable but a very weak requirement. It does not account at all for distributional aspects.
- Government intervention may be particularly desirable if the assumptions of the 1st welfare theorem fail (i.e., whenever there are the so-called «market failures».
  - Government intervention can potentially improve everybody's welfare.
- Moreover, one could ask if the reverse of the 1st Theorem of Welfare Economics is also valid
  - Is it possible to obtain a given Pareto efficient allocation as a market equilibrium?



- Governments may want to redistribute the initial resources from point W to point W'
- Such a redistribution will be "appropriate", since the budget constraint passing through W' also passes through M', as long as:
  - Individual indifference curves are "regular", i.e. convex;
  - Redistribution is not distortionary

2nd Theorem of Welfare Economics

Theorem

If the individuals' preferences are convex, then every Pareto-efficient allocation can be obtained as a market equilibrium starting from an opportune and not distortionary redistribution of the initial endowements

- 1st Theorem of Welfare Economics says that market allocations are (generally) efficient.
- There are potentially infintely many of such allocations. An equity principle may help to select one of them, depends on the preference of the society as a whole.
- 2nd Theorem of Welfare Economics says that each of these efficient allocations is achieveable through a non distortionary redistribution of the initial resources by governments, thus the social optimum will be both efficient and fair.

$\Rightarrow$ **Governments may intervene to redistribute resources**

Fallacy of 2nd Welfare Theorem

- In reality, 2nd welfare theorem does not work because redistribution of initial endowments is not feasible (because initial endowments cannot be observed by the government)
  - Governments need to use **distortionary** taxes and transfers based on economic outcomes (such as income or working situation)
  - Conflict between efficiency and equity: **Equity-Efficiency trade-off**

Illustration of 2nd Welfare Theorem Fallacy

- Suppose economy is populated 50% with disabled people unable to work (hence they earn \$0) and 50% with able people who can work and earn \$100
- **Free market outcome:** disabled have \$0, able have \$100
- **2nd welfare theorem:** government is able to tell apart the disabled from the able (even if the able do not work)
  - Can tax the able by \$50 [regardless of whether they work or not] to give \$50 to each disabled person $\Rightarrow$ the able keep working (otherwise they'd have zero income and still have to pay \$50)
- **Real world:** govt can't tell apart disabled from non working able
  - \$50 tax on workers + \$50 transfer on non workers destroys all incentives to work $\Rightarrow$ govt can no longer do full redistribution $\Rightarrow$ Trade-off between equity and size of the pie

Social welfare functions

- Economists incorporate distributional aspects using **social welfare functions** (instead of just adding $ of economic surplus)
- **Social welfare function (SWF)**: A function that combines the utility functions of all individuals into an overall social utility function

$$W = W(U_A, U_B) \tag{1}$$

  - Once defined a SWF, we can rank all possible social states according to a society's preferences for equity vs. efficiency.
  - It represents the judgement of the society (and who rule it) on the distribution of utility between its members
- The general idea of social welfare functions is that one dollar to a disadvantaged person might count more for the society than one dollar to a riche person

Utilitarian social welfare function

- With a utilitarian social welfare function, society's goal is to maximize the sum of individual utilities:

$$SWF = U_1 + U_2 + ... + U_N$$

- The utilities of all individuals are given equal weight, and summed to get total social welfare
- Utilitarian criterion still values redistribution from rich to poor because of decreasing marginal utility of consumption
  - One extra unit of consumption for individuals already consuming more is valued less than one extra unit of consumption for individuals who do not consume equally much.
- Taking \$1 from a rich person decreases his utility by a small amount, giving the \$1 to a poor person increases his utility by a large amount
  $\Rightarrow$ Transfers from rich to poor may increase total utility

Rawlsian social welfare function

- Rawls (1971) proposed that society's goal should be to maximize the well-being of its worst-off member. The Rawlsian SWF has the form:

$$SWF = \min\{U_1, U_2, ..., U_N\}$$

- Since social welfare is determined by the minimum utility in society, social welfare is maximized by maximizing the well-being of the worst-off person in society (=maxi-min)
- Rawlsian criterion is even more redistributive than utilitarian criterion: society wants to extract as much tax revenue as possible from the middle and rich to make transfers to the poor as large as possible.

Generic social welfare functions

- A sufficient statistics that captures the social preferences for equity vs. efficiency can be obtained with a following SWF

$$SWF = \sum_{i=1}^{N} \omega_i U_i$$

- where ω_i represents the (arbitrarily chosen) social weight that the society puts on individual i's utility.
- Two extreme cases of such SWF are the utilitarian and the rawlsian SWFs
  - $\omega_i = 1 \; \forall \; i \; \rightarrow$ utilitarian SWF
  - $\omega_i = 1$ if $U_i = min\{U_1, U_2, ..., U_N\}$ and $\omega_i = 0$ if $U_i \neq min\{U_1, U_2, ..., U_N\} \; \rightarrow$ rawlsian SWF
- A way to show off strong but not extreme preferences for redistribution is to set up ω_i to be inversely proportional to one individual's income (or consumption, or utility)

Other social justice principles

- Standard welfarist approach is based on individual utilities. This fails to capture important elements of actual debates on redistribution and fairness
  1. **Just deserts**: Individuals should receive compensation congruent with their contributions (libertarian)
     $\Rightarrow$ Taxes should be tailored to government benefits received
  2. **Commodity egalitarianism**: Society should ensure that individuals meet a set of basic needs (seen as rights) but that beyond that point income distribution is irrelevant
     $\Rightarrow$ Rich countries today consider free education, universal health care, retirement/disability benefits as rights
  3. **Equality of opportunity**: Society should ensure that all individuals have equal opportunities for success
     $\Rightarrow$ Individuals should be compensated for inequalities they are not responsible for (e.g., family background, inheritance, intrinsic ability) but not for inequalities they are responsible for (being hard working vs. loving leisure)

An empirical test of people's social preferences

- Saez-Stantcheva (2016) surveyed people online (using Amazon MTurk) by asking hypothetical questions to elicit social preferences. Key findings:
  1. People typically do not have "utilitarian" social justice principles (consumption lover not seen as more deserving than frugal person)
  2. People put weight on whether income has been earned through effort vs. not (hard working vs. leisure lover)
  3. People put a lot of weight of what people would have done absent the government intervention (deserving poor vs. free loaders)

Actual social preferences

- **General conclusion:** People favor redistribution if they feel inequalities are "unfair" but views on what is fair differ

  $\Rightarrow$ Redistribution supported when people don't have control (education for children, health insurance for the sick, retirement/disability benefits for the elderly/disabled unable to work)

  $\Rightarrow$ Less support when people have some or full control (unemployment, being low income)

  $\Rightarrow$ Less support when people don't "belong" (us vs. them)

- Conservatives (Republicans) tend to frame things: individuals have control (personal responsibility), govt should just enforce rules
- Liberals (Democratic) tend to frame things: many forces in society beyond individuals' control ("we are all in this together"), society should provide nurturing

To conclude: two general rules for government intervention

1. **Market Failures:** Government intervention can help if there are market failures
2. **Redistribution:** Free market generates inequality. Govt taxes and spending can reduce inequality

- Who decides how redistribute (e.g., dictatorship vs democracy / constitution vs elections / good vs. bad politician)
  - What does it happen when the government fails?

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