Political Economics
Problem Set 3

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November 23, 2017

Note

The solution to this problem set should be uploaded to Studentportalen no later than **December 5** at 24:00. Write your individual answers on computer and put your name at the top of the document. This problem set is about rents for politicians, agency models and Institutions. Please carefully motivate your answers. For any questions concerning the exercises, send me an e-mail at davide.cipullo@nek.uu.se. My office is **E434** at the Department of Economics. Good luck!

1. **Rents for politicians.** Consider voters who have the following quasi-linear utility function: $U_i = c_i + G^{\frac{1}{2}}$, where c_i is private consumption and G is a public good. There are two politicians, Jack and Susan (henceforth J and S), who run for office. They have linear expected utility function: $U_p = P[R+r_p]$, $p \in \{J, S\}$ where P is the probability of winning the election. $R > 0$ is an exogenous ego-rent from being in office, while $r > 0$ represents endogenous rent that an elected politician can extract from the public budget. Voters' budget constraint is $c_i = (1-t)w_i$, where w_i is private income. Politicians do not know individual wages, but they know that wages are distributed with expected value $E(w_i) = \bar{w}$. The government budget constraint is $t\bar{w} = G + r$.

   Candidates announce policy platforms in terms of t and G, before the elections. The winner is committed to implement the preferred policy.

   (a) What level of rents r will candidates announce prior to the election to maximize expected utility?

   The unique way to get the exogenous rent $R > 0$ is to be in office. Hence, candidates need to maximize the probability of victory to maximize their expected utility. Therefore, they will surely announce $r = 0$.

   (b) Derive the sign of $\frac{\partial U(\cdot)}{\partial r}$ for consumers and explain the economic intuition.

   Let us substitute the private constraint and the government constraint into the individual objective function. Hence:

   Derivation of the partial derivative

$$U_i(t, r) = (1-t)w_i + (t\bar{w} - r)^{\frac{1}{2}}$$

   Thus,

$$\frac{\partial U_i}{\partial r} = -\frac{1}{2}(t\bar{w} - r)^{-\frac{1}{2}}$$

   we observe that $(t\bar{w} - r)^{-\frac{1}{2}} = G^{-\frac{1}{2}} > 0$. Hence, $\frac{\partial U_i}{\partial r} < 0$.

   Derivation of the partial derivative

(c) Derive consumers' preferred tax rate as a function of w_i and $\bar{w}$.

We start with a trick: since we just derived that $\frac{\partial U_i}{\partial r} < 0$, it is always optimal from the individual perspective to have $r = 0$. Hence, we can solve the problem in the case in which $r = 0$. Let us start from the utility function above:

Solution to consumer's problem

$$U_i(t) = (1-t)w_i + (t\bar{w})^{\frac{1}{2}}$$

Hence, FOC are:

$$\frac{\partial U_i}{\partial t} = -w_i + \frac{\bar{w}}{2}(t\bar{w})^{-\frac{1}{2}} = 0$$

Rearranging:

$$\frac{\bar{w}}{2}(t\bar{w})^{-\frac{1}{2}} = \frac{2w_i}{\bar{w}}$$

$$t\bar{w} = (\frac{2w_i}{\bar{w}})^{-2}$$

Therefore,

$$t = \frac{1}{\bar{w}}(\frac{2w_i}{\bar{w}})^{-2}$$

or, equivalently,

$$t = \frac{\bar{w}}{4w_i^2}$$

(d) Discuss why demand for taxation depends on the wage relative to the average.

(e) Suppose instead that politicians may not commit. Do their proposals prior to the election change? Does the policy implemented by the winner changes? No maths is required but carefully explain the economic intuition.

(f) Discuss briefly the link between commitment and rent extraction.

2. **Agency Models.** Consider an agency problem with two time periods with one election in between. Voters are homogeneous, with utility function $U_{i,t} = ln(G_t)$, where G_t is public good spending at time t. There are two types of politician: one good politician (henceforth 1) with utility function $U_{1,t} = ln(G_t)$, and one rent-seeking politician (henceforth 2) with utility function $U_{2,t} = G_t^{\alpha} r_t^{1-\alpha}$, where r_t represents an endogenous ego-rent that the politician may extract when in office. The government budget constraint is $\tau = G_t + \delta_t r_t$, where δ_t is a random variable (whose expected value is $E(\delta)$ representing the cost associated with rent extraction in time period t, and τ is exogenous tax revenues of the government. Timing is the following: Nature picks an incumbent politician from a pool of politicians. He or she is good (ρ_1) with probability μ and rent-seeker (ρ_2) with probability $1-\mu$, where $0 < \mu < 1$. The incumbent observes his or her own type and δ_t, that are hidden from the voters, that only know the expected value $E(\delta)$. Then, the incumbent politician implements a policy according to her type. Voters observe the implemented policy and update their beliefs about the incumbent's type.
The election takes place and voters make decision according to their updated belief. After the election, δ_t for time period 2 is realized, and the winner implements a policy for the second period. Throughout the exercise, assume that politicians are risk-neutral.

(a) Which policy will each type of politician implement in the last time period?

In the last time period, each type of politician, if elected, will implement her preferred policy. Therefore, the good politician would implement $G_2 = \tau$, while the bad politician will choose his policy according to the following optimization problem:

Solution to the bad politician's problem

$$\max_{r_2, G_2} \quad U_{2,2}(r_2, G_2) = G_2^{\alpha} r_2^{1-\alpha}$$
$$\text{subject to} \quad \tau = G_2 + \delta_2 r_2$$

We plug the constraint into the objective function in order to remain with only one free variable left.

$$\max_{r_2} \quad U_{2,2}(r_2) = (\tau - \delta_2 r_2)^{\alpha} r_2^{1-\alpha}$$

We do a logarithmic transformation so that calculus becomes easier.

$$\max_{r_2} \quad ln\big(U_{2,2}(r_2)\big) = \alpha ln(\tau - \delta_2 r_2) + (1-\alpha) ln r_2$$

Therefore, the FOC are:

$$\frac{\partial U_{2,2}}{\partial r_2} = -\frac{\alpha}{\tau - \delta_2 r_2}\delta_2 + \frac{1-\alpha}{r_2} = 0$$

Rearranging

$$\alpha \delta_2 r_2 = (1-\alpha)(\tau - \delta_2 r_2)$$

or

$$\alpha \delta_2 r_2 = \tau - \delta_2 r_2 - \alpha\tau + \alpha \delta_2 r_2$$

Solving for r_2:

$$r_2 = \frac{\tau(1-\alpha)}{\delta_2} > 0$$

Solution to the bad politician's problem

(b) Assume that in the first time period the rent-seeking politician is in office, and that voters will re-elect a politician if she implements their preferred policy. Derive the value of δ_2 that makes the rent-seeking politician indifferent between extracting rents in the first period and pooling with the good politician.

If voters always re-elect the incumbent that implement their preferred policy in the first period, the rent-seeking politician maximizes his rent implementing $G_1 = \tau$ if what he expects to receive in the second period remaining in office exceeds the maximum rent that he can exploit in the first period (and then losing elections). Then, what is to be taken into account is that the corrupt politician will implement a policy that is different from $G_1 = \tau$ if and only if the rent he is able to exploit in the first period is more than what he is expected to exploit in the second period if he mimic a good politician in the first period. Of course, the honest politician will always implement $G_1 = \tau$. Therefore, the bad politician is indifferent between extracting rents in $t = 1$ and pooling with the good politician if and only if $0 + r_2 = r_1 + 0$. The equation implies that if the rent-seeking politician extracts rent in the first period, she will not be re-elected. We have already shown that $r_2 = \frac{\tau(1-\alpha)}{\delta_2}$. With the same procedure we also obtain $r_2 = \frac{\tau(1-\alpha)}{\delta_1}$. Therefore, the condition becomes $\frac{\tau(1-\alpha)}{\delta_1} = \frac{\tau(1-\alpha)}{\delta_2}$, which implies $\delta_1 = \delta_2$. However, the timing of the problem is such that the incumbent politician does not know the value δ_2 when she makes the first decision Hence, she uses $E(\delta)$ instead of δ_2. Therefore, the rent-seeking politician is indifferent between the two strategies if and only if $\delta_1 = E(\delta)$.

(c) In the previous point we assumed that voters will re-elect a politician if she implements their preferred policy. Let $\theta \in [0,1]$ the probability that the bad politician will pool with the good one in the first round.

(d) During the elections, the incumbent faces an opponent drawn at random by Nature. Discuss in words under which condition voters would support the incumbent against the random challenger.

Voters will support the incumbent during the elections if the posterior probability of having a good politician as incumbent is higher than μ. This is because they know that in the last period a good politician will always implement $G_2 = \tau$, while the bad politician will always extract positive rent r_2 as derived before.

(e) Use Bayes' rule to derive whether to support the incumbent that implemented $G_1 = \tau$ is consistent with voters' belief at the time of the elections. In this situation, the Bayes' rule has the following form:

Application of the Bayes' rule

$$P(\rho_1|G_1 = \tau) = \frac{P(\rho_1)P(G_1 = \tau|\rho_1)}{P(G_1 = \tau)}$$

Application of the Bayes' rule

i.e. the probability that the incumbent is honest given that he implemented the policy $G_1 = \tau$ is equal to the prior probability of having a honest politician times the probability of observing $G_1 = \tau$ in the presence of a good incumbent divided by the probability of observing $G_1 = \tau$ whatever is the incumbent's type. But $P(\rho_1) = \mu$, $P(G_1 = \tau|\rho_1) = 1$ and $P(G_1 = \tau) = \mu + (1-\mu)\theta$, then the probability of a honest incumbent after observing $G_1 = \tau$ is

$$P(\rho_1|G_1 = \tau) = \frac{\mu}{\mu + (1-\mu)\theta} > \mu$$

if $\theta < 1$. Therefore, when voters evaluate new beliefs according to the Bayes' rule, if they observe $G_1 = \tau$ their updated belief on the incumbent is higher than the prior probability to have a good politician. This implies that to always vote for the incumbent who have set $G_1 = \tau$ is a consistent strategy for voters, since the probability that the incumbent politician is a honest one is higher than the probability that the random guy that is running in the election as opponent is a honest one. This implies that for voters is "optimal" to trust the corrupt politician that behaved as honest before.

(f) Use Bayes' rule to show that to re-elect the incumbent who did not implement $G_1 = \tau$ is never consistent with voters' belief. In this situation, the Bayes' rule has the following form:

Application of the Bayes' rule

$$P(\rho_1|G_1 < \tau) = \frac{P(\rho_1)P(G_1 < \tau|\rho_1)}{P(G_1 < \tau)}$$

Application of the Bayes' rule

i.e. the probability that the incumbent is honest given that he implemented the policy $G_1 < \tau$ is equal to the prior probability of having a honest politician times the probability of observing $G_1 < \tau$ in the presence of a good incumbent divided by the probability of observing $G_1 < \tau$ whatever is the incumbent's type. But $P(\rho_1) = \mu$, $P(G_1 < \tau|\rho_1) = 0$ and $P(G_1 = \tau) = (1-\mu)$, then the probability of a honest incumbent after observing $G_1 < \tau$ is

$$P(\rho_1|G_1 < \tau) = \frac{0}{\mu + (1-\mu)} = 0 < \mu$$

(g) Assume instead that the good politician is drawn by Nature. Will she have the chance to reveal with certainty her type to voters? Discuss (no maths is required).

> No. We have already seen in (b) that there are some conditions under which there is a pooling equilibrium. Therefore, under those conditions, the good politician cannot signal her type with certainty.

(h) Consider an alternative version of the model. Now the government budget constraint is $\tau = \delta_t(G_t + r_t)$, where δ_t now is a random variable that represents a loss of resources that the incumbent politician has to face to finance her policies. All other assumptions are as in the text. Discuss why under this budget constraint the rent-seeking politician might potentially extract rents during both periods.

(i) Consider now a three-periods model, with two elections in the middle. Apart for that, consider the same assumptions stated in the main text of the problem. Assume also that in the first time period the rent-seeking politician is in office, and that voters will re-elect a politician if she implements their preferred policy. Derive the value of δ_t that makes the rent-seeking politician indifferent between extracting rents in the first period and pooling with the good politician. Can we conclude that in this case the rent-seeking politician is now better-off compared to (a)?

> In the third period, each politician will implement her preferred policy. Specifically the rent-seeking one will implement according to the calculus from above $r_3 = \frac{\tau(1-\alpha)}{\delta_3} > 0$. In the second period, she will be indifferent between extracting rents and pooling with the good politician if and only if $\delta_2 = E(\delta)$. In the first period, she will be indifferent between extracting rents and pooling with the good politician if and only if $\delta_1 = E(\delta)$. Therefore, the rent-seeking politician is neither better-off nor worse-off compared to the two-periods model.

3. **Institutions.** Answer the following questions in a short paragraph each.

(a) Provide an economic intuition on why majoritarian and proportional voting rules lead to different economic policies.

(b) Make one real-world example of inclusive institution and one of extractive institution in ancient history. Briefly compare economic performances back in time and nowadays between the two localities.