Political Economics
Problem Set 1

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Note

The solution to this problem set should be uploaded to Studentportalen no later than **November 9** at 24:00. Write your individual answers on computer and put your name at the top of the document. This problem set is about Social choice, voting and bargaining. Please carefully motivate your answers. For any questions concerning the exercises, send me an e-mail at davide.cipullo@nek.uu.se. My office is **E434** at the Department of Economics. Good luck!

1. **Social choice.** According to Arrow's Impossibility Theorem, it is not possible for a society to aggregate individual preferences in a Social Welfare Function that satisfies simultaneously the three axioms you have been introduced to in class (Universality of domain; Weak Pareto principle and Independence of irrelevant alternatives).

   (a) Was the statement correct and complete? Motivate your answer in one short paragraph.

   (b) Formally define the IIA axiom. Provide an intuition on and explain why the majoritarian voting rule does not satisfy it in general if there are at least three choices.

2. **Social choice.** In this question, you will go through an application of real-world voting rules on sport competition.

   Consider a pentathlon contest in which four athletes have to compete in the following disciplines: shooting, swimming, fencing, high-jumping and cross-country running. In shooting, contestants have to hit 50 marks. In swimming, they have to swim 50m as fast as possible. In fencing, they play a tournament with semi-finals and finals. In high-jumping, they have to jump over obstacles posed at an increasing height. In cross-country running, they have to run 20km.

   Look at results in Table 1.

Table 1: Results of the pentathlon competition

Athlete	Shooting	Swimming	Fencing	High-Jumping	Running
A	43/50	1:00	#4	2.30 m	1:00:35
B	38/50	0:56	#2	2.24 m	0:58:37
C	45/50	0:55	#3	2.00 m	1:00:00
D	21/50	0:48	#1	2.09 m	1:02:40

Disciplines are different and hard to compare. In this task, you are the director of the pentathlon, and want to decide a classification mechanism. For this purpose, you make some comparisons between decision-making mechanisms:

(a) The winner of the pentathlon is the athlete who wins the largest number of disciplines. Who wins the pentathlon in this case? Explain your reasoning.

(b) If one of the athletes wins in more than half of disciplines, she wins the pentathlon. Otherwise, you compare, among the two contestants with the largest number of victories (or of other positioning, in case of a tie), their relative ranking in all five disciplines. Who wins the pentathlon in this case? Explain your reasoning.

(c) You are now allowed to give, for each discipline, one point to the third classified, two points to the second classified and three points to the first, and to sum results from the five disciplines. Then, you exclude the athlete with the worst score. Among the three not excluded, you assign for each discipline one point to the second classified, and two points to the first. You exclude the athlete with the worst score. Among the two remaining, the winner is the one that classifies better than the other at least in three disciplines. Who wins the pentathlon in this case? Explain your reasoning and report the tables with the scores. *Hint: Every time you exclude an athlete, reset the score to zero for all remainders.*

(d) The three decision-making procedures described above are examples of three majoritarian voting rules implemented in certain countries for national elections. The first one is the *plurality rule* (also known as *first-past-the-post*), the second one is the *runoff system* (also known as *dual-ballot rule*), and the third one is known as *elimination runoff*. Give an intuition about why they can potentially lead to different winners in the electoral competition, and report, for each of them, an empirical example of country in which the system is implemented. What is the minimum number of available choices to make each of them admit a different winner?

(e) Compare your answers from (b) and (c) in view of the Condorcet Voting paradox. Limit your answer to one page.

3. **Voting.** Consider an economy of five individuals $i = \{1, 2, 3, 4, 5\}$ that make decisions on how much to invest in public education. All individuals have the same quasi-linear utility function: $U_i(c_i, G) = c_i + ln(G)$ where c is a consumption (private) good, and G is the public good (education). The public good is financed through a proportional income tax t. Hence, the government budget constraint is $G = \sum_{i=1}^{5} tw_i$ while individual budget constraint is $c_i = (1 - t)w_i \ \forall i \in \{1, ..5\}$. Wages are exogenously given: $w_1 = w_2 = 1$; $w_3 = 2$; $w_4 = 3$; $w_5 = 5$.

(a) Who is the median voter in this society? Do the assumptions of the Median Voter Theorem apply? Discuss.

(b) What is the expenditure on education that the society will choose? What is the tax rate? Solve the model.

(c) What would have been the expenditure in education and the tax rate in the case of a benevolent central planner that maximizes an utilitarian SWF? Solve the model.

Suppose now that all 5 individuals have equal wage $w = 2$, but preferences are instead represented by the utility function $U_i(c_i, G) = c_i^{i/5} G^{(5-i)/5} \ \forall i \in \{1, ..5\}$. The government budget constraint and the individual budget constraints are the same as before.

(d) What is the expenditure in education that the society will choose? What is the tax rate? Solve the model.

(e) What is the preferred tax rate of individual 5? Carefully explain in words and graphically the economic intuition.

(f) Compare results from points (b) and (c). Limit your answer to half page.

4. **Bargaining in Legislature.** In a parliamentary democracy, three parties are in the House of Representatives. We refer to them as S (*Social-democrats*), C (*Conservatives*) and P (*Populists*). None of them got the majority during the elections, but a coalition of two is necessary and sufficient to form a government. Parties can bargain over the number of ministries assigned to each party s_i, over taxation rates ($t \in [0, 1]$), or over both dimensions. A government must be composed by 5 ministries and all parties of the coalition must be represented with at least one minister. S and C are both policy and office motivated, with utility function $U_i = m_i - (a_i - 3t)^2$ where m_i is the number of ministries assigned to party i and a_i is a parameter measuring the preference for taxation, such that $a_i = \frac{3}{2}$ for S and $a_i = 1$ for C. On the other hand, the populist party P values less the number of ministries, so its utility function is $U_P = \frac{1}{3}m_P - (1 - 3t)^2$. If either S or C is indifferent between making the coalition with the other, or making it with P, we assume that it will not choose P.

The distribution of seats is the following: $w_C > w_S > w_P$. The party with the largest share is allowed to make the first proposal. If a coalition is formed, parties receive payoffs accordingly. Otherwise, the party with the second seat share make a proposal. If a coalition is formed, parties receive payoffs accordingly. If no coalition is made, the third party makes a proposal. If again no coalition is formed, all parties get an exogenous payoff of 0.

(a) What is the preferred tax rate of each party?

(b) Solve for the equilibrium tax rate and ministry allocation. Which parties will form the governing coalition? Can you assess at which round the proposal will be accepted?

(c) Now assume that all parties have utility function $U_i = m_i - (\frac{1}{3} - 3t)^2$ and that a coalition can be formed even if one of the members receives 0 ministers. Solve for the equilibrium tax rate and ministry allocation. Compare with (b).

(d) Discuss your results in relation to agenda setting power. Limit your answer to half page.